South Carolina College- and Career-Ready Standards for Mathematics

7th Grade Support Document

South Carolina Department of Education
Office of Standards and Learning
January 2016 - DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Grade 7 Overview

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for middle school into *Course Coversheets* and *Units*.

- Each middle school *Course Coversheet* organizes the middle school course standards into possible instructional units and provides links to specific middle school course *Units*.
- Each middle school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Potential instructional strategies and lessons
  - Resources for the unit
  - Sample formative assessment tasks and questions

South Carolina College- and Career-Ready Standards for Mathematics
Grade 7 Overview

Table of Contents

Table of Contents (Continued)

South Carolina College- and Career-Ready Standards for Mathematics
Middle School Support Document
Grade 7 Coversheet

Return to Middle School Overview or Table of Contents

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Rational Numbers	Expressions and Equations	Ratios and Proportions	Geometry	Algebraic Geometry	Statistics	Probability
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
7.NS.1	7.EEI.1	7.RP.1	7.GM.1	7.GM.4	7.DSP.1	7.DSP.5
7.NS.2	7.EEI.2	7.RP.2	7.GM.2	7.GM.5	7.DSP.2	7.DSP.6
7.NS.3	7.EEI.3	7.RP.3	7.GM.3	7.GM.6	7.DSP.3	7.DSP.7
	7.EEI.4	7.NS.5		7.EEI.4b	7.DSP.4	7.DSP.8
	7.EEI.5	7.EEI.4				
	7.NS.4	7.GM.1				

Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

a. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

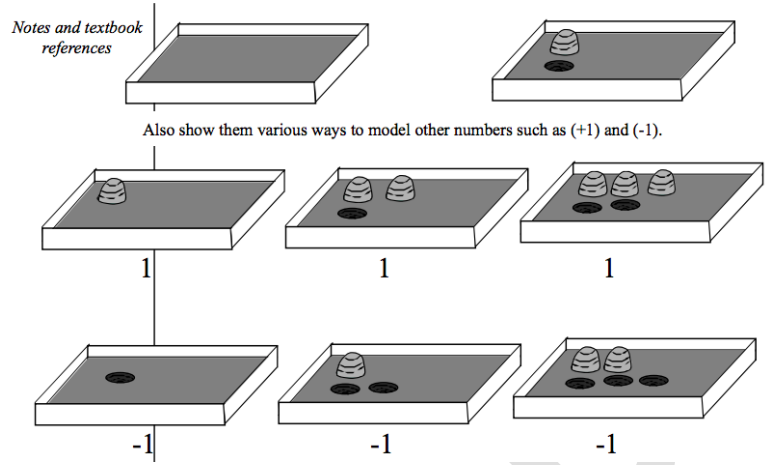
Unit Title
Rational Numbers
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.NS.1 Extend prior knowledge of operations with positive rational numbers to add and to subtract all rational numbers and represent the sum or difference on a number line. a. Understand that the additive inverse of a number is its opposite and their sum is equal to zero. b. Understand that the sum of two rational numbers ($p + q$) represents a distance from p on the number line equal to $\|q\|$ where the direction is indicated by the sign of q. c. Translate between the subtraction of rational numbers and addition using the additive inverse, $p - q = p + (-q)$. d. Demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference. e. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to add and subtract rational numbers. o Understand, apply, and explain the additive inverse o Model addition and subtraction of rational numbers, including integers, decimals, and fractions using visual models **7.NS.2** Extend prior knowledge of operations with positive rational numbers to multiply and to divide all rational numbers. a. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one. b. Understand sign rules for multiplying rational numbers. c. Understand sign rules for dividing rational numbers and that a quotient of integers (with a non-zero divisor) is a rational number. d. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to multiply and divide rational numbers. e. Understand that some rational numbers can be written as integers and all rational numbers can be written as fractions or decimal numbers that terminate or repeat. o Understand, apply, and explain the multiplicative inverse o Model multiplication and division of rational numbers, including integers, decimals, and fractions using visual models o Explain why division by zero is undefined **7.NS.3** Apply the concepts of all four operations with rational numbers to solve real-world and mathematical problems. o Model and solve real-world problems using numbers and operations o Explain the solution to a real-world problem in the context of the problem o Interpret the reasonableness of solutions in the context of the problem

New Academic Vocabulary for This Unit
● Integer ● Positive ● Negative ● Additive inverse ● Zero pairs ● Multiplicative inverse ● Terminating decimal ● Repeating decimal

Prior Knowledge Required for this Unit
● Fluent with operations of positive rational numbers (4.NSBT.4, 5.NSBT.5, 5.NSF.1, 5.NSF.4, 6.NS.1, 6.NS.2, 6.NS.3) ● Understand and perform operations using absolute value (6.NS.7) ● Fluent understanding of mathematical properties (e.g., commutative associative, distributive, or the properties of identity and inverse elements)(3.ATO.5, 4.NSBT.6, 5.NSBT.6, 6.EEI.3, 6.EEI.4) ● Fluent with the Order of Operations with positive rational numbers (3.ATO.8, 4.ATO.3, 5.ATO.1, 6.EEI.1, 6.EEI.2)
Subsequent Knowledge Related to this Unit
This unit will lead to mastery of the Order of Operations involving the fraction bar as a grouping symbol with integers in Grade 7. These standards will also guide students when solving expressions, equations, and inequalities with rational coefficients. The introduction of terminating and repeating decimals will extend to an understanding of 7.NS.5 involving the translation among multiple representations of rational numbers excluding a repeating decimal to a fraction. In Grade 8, students will continue to solve linear equations and inequalities with rational numbers; mastery of these skills will lay a strong foundation for success in high school math courses. In Grade 8, students will extend their knowledge to multiple representations of rational and irrational numbers.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop computational skills necessary for work with rational numbers.

Potential Instructional Strategies/Lessons

- Visual models
  - Two color counters
  - Addition and Subtraction
    - Heaps and Holes: Explain to your students that +1 is like a pile (or heap) of sand on a level beach. A hole of equivalent size dug into the beach represents -1. This model explains positives as a surplus and negatives as a deficit. (See page 9)



Source: NC Grade 7 Classroom Strategies

  - Multiplication

Source: Brain Camp

  - Number line (horizontal and vertical)

- Tracking Text: This strategy is used to process a word problem. The process entails underlining essential information needed to solve the given problem, including the labels as well as circling important math terms and renaming them with known vocabulary to assist in understanding what process to follow when solving for the answer.

  Example: When the navy wants to test the depth of a submarine they complete the following steps. First they take the submarine to a depth of 150 meters below sea level followed by moving up towards the surface 100 meters. The final step is to go deeper another 350 meters from where they were. How deep will the submarine be after this series of movements under water?

  negative
  positive
  negative

  -150m + 100m + (– 350m)

Resources
6–8 Progression on The Number System; High School, Number - http://commoncoretools.me/wp-content/uploads/2013/07/ccssm_progression_NS+Number_2013-07-09.pdf *Elementary and Middle School Mathematics: Teaching Developmentally*, Eighth Edition (older editions can be used as well) By John A. Van de Walle, Karen S. Karp, and Jennifer M. Bay-Williams ISBN-10: 0132612267; ISBN-13: 9780132612265 7.NS.1 - Through this unit, students will be able to analyze the addition and subtraction of integers by discussing the rise and fall of a hot air balloon. http://www.supermathunits.com/files/hot_air_ballon_unit.pdf 7.NS.2 - This document provides station-based activities for the classroom to assist students with the multiplication and division of rational numbers. http://moodle.wbrschools.net/pluginfile.php/3830/mod_resource/content/1/Set%202%20Multiplying%20and%20Dividing%20Rational%20Numbers.pdf 7.NS.1- 7.NS.3 - This Jeopardy game provides students with a computational review of operations with rational numbers. http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html
Sample Formative Assessment Tasks/Questions
Performance Task 7.NS.1: Jennifer decided to open a bank account with $150. On Monday she made a deposit of $25. The following week she made a withdrawal of $150. The next month there was an emergency and she needed to withdraw $200. After this last withdrawal does Jennifer have a positive or negative balance? Explain. Answer: Jennifer would have a negative balance in her account after the last withdrawal. You find this out by adding $25 to $150 to get a balance of $175. You then subtract the withdrawal of $150 from $175 to get a balance of $25. Finally you would subtract $200 from $25 to get a negative balance of -$175.

7.NS.2: A water well drilling rig has dug 60 feet below the surface of the water after one full day (24 hrs) of continuous use. A. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours? Answer: -37.5 feet B. If the rig has been running constantly and is currently at a height of -143.6 feet, for how long has the rig been running? Answer: approximately 2.93 hours
7.NS.2: The cheerleaders at a local middle school want new uniforms for the yearly cheerleading competition in South Carolina. A parent donated 20 ¾ yards of material. If each uniform takes ⅞ yard to make, how many uniforms can they make from the donated material? Explain your reasoning. Answer: 23 uniforms - If the total yardage is 20 ¾ yards and each uniform requires ⅞ yard, you must divide 20 ¾ by ⅞. The answer is 23 5/7 but because we are talking about uniforms, only 23 uniforms can be made from the material.
7.NS.3: Justin has two dogs, Finley and Noni. Each day Finley eats ⅓ of a can of dog food, and Noni eats ⅘ of a can. Dog food is only sold in 4 can packages, and each package costs $3. How much will Justin pay for a 30 day supply of dog food? Justify your reasoning. Answer: $27 - First, students must determine how many cans of dog food Justin will need for 30 days. By multiplying ⅓ of a can by 30 days, we see Finley requires 10 cans of dog food. By doing the same with ⅘ of a can and 30 days, we see Noni needs 24 cans of dog food. This means Justin will need 34 total cans of dog food for 30 days. If he can only buy cans of dog food in packs of 4, students must divide 4 into 34 to determine how many packages Justin must buy. The resulting quotient is 8 ½; however, individuals cannot buy partial packages of dog food. Therefore, Justin must buy 9 packages of dog food at $3 per package for a total price of $27.
7.NS.3: During this task, students will solve real-world and mathematical problems involving the four operations with rational numbers. They will also translate those operations into words/stories.  7.NS.3 Lesson Iced Tea Stand.doc
7.NS.3: A 20-foot piece of rope will be cut into as many 1 ½ foot sections as possible. How much rope will be left over? Answer: 6 inches; ½ foot

Unit Title
Expressions and Equations
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.EEI.1 Apply mathematical properties (e.g., commutative, associative, distributive) to simplify and to factor linear algebraic expressions with rational coefficients. ○ Use properties to simplify and factor algebraic expressions, including those with rational coefficients ○ Use the greatest common factor to factor algebraic expressions ○ Foundational formulas (e.g., perimeter, area) should be used to extend knowledge of algebraic expressions **7.EEI.2** Recognize that algebraic expressions may have a variety of equivalent forms and determine an appropriate form for a given real-world situation. ○ Describe real-world situations and determine the appropriate form of an algebraic expression (e.g., 2(x+4) or 2x+8) **7.EEI.3** Extend previous understanding of Order of Operations to solve multi-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol. ○ Understand that the fraction bar implies parentheses for the numerator ○ Order of Operations should include multiplication and division of rational numbers, including integers, decimals, and fractions **7.EEI.4** Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations. a. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where a, b, and c are rational numbers. b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides. c. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning. d. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities. ○ Equations must remain balanced ○ Use Order of Operations in the reverse order to isolate the variable **7.EEI.5** Understand and apply the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property) to simplify numerical expressions that include whole-number exponents. ○ Understand that a number raised to a power of zero results in a value of 1, not 0 ○ Derive the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property) before applying the laws in numerical expressions

7.NS.4 Understand and apply the concepts of comparing and ordering to rational numbers.

a. Interpret statements using less than (<), greater than (>), less than or equal to (≤), greater than or equal to (≥), and equal to (=) as relative locations on the number line.
b. Use concepts of equality and inequality to write and explain real-world and mathematical situations.
   - Understand that a closed circle represents the rational number and an open circle will not represent the rational number when graphing an inequality
   - Focus on real-world situations including vocabulary for less than or equal to (≤) and greater than or equal to (≥)

New Academic Vocabulary for This Unit
• Product Rule • Quotient Rule • Power to a Power • Product to a Power • Quotient to a Power • Zero Power Property • Greater than or Equal to • Less than or Equal to

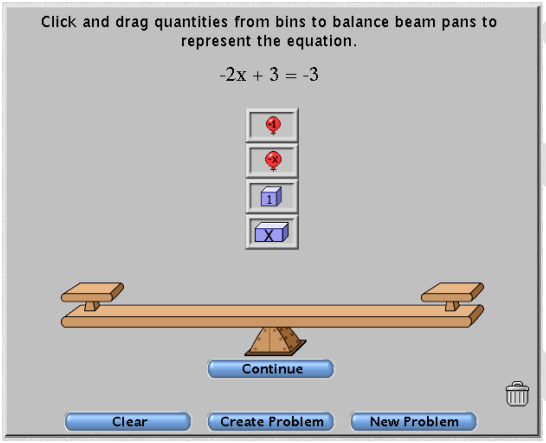
Prior Knowledge Required for this Unit
• Evaluate numerical expressions involving grouping symbols (5.ATO.1) • Write and evaluate numerical and algebraic expressions with whole number exponents and positive rational numbers (6.EEI.1, 6.EEI.2) • Apply mathematical properties to generate and justify equivalent expressions (6.EEI.3, 6.EEI.4) • Calculate the greatest common factor of two numbers less than or equal to 100 (6.NS.4)
Subsequent Knowledge Related to this Unit
In Grade 8, students will extend knowledge of the Laws of Exponents to include numerical expressions with integer exponents. In high school courses, students will apply the Laws of Exponents with rational exponents and algebraic expressions. Additionally in Grade 8, students will solve multi-step equations with variables on both sides of the equal sign. Grade 7 students will use their complete understanding of the Order of Operations and their properties to lay a foundation for functions in Grade 8.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop the computational skills needed for work within the real number system including solving multi-step linear equations and inequalities and simplifying expressions which include the use of whole-number exponents.

Potential Instructional Strategies/Lessons

- Algebra Tiles

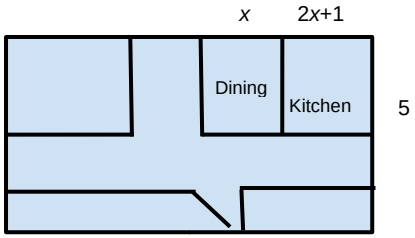
Source: National Library of Virtual Manipulatives

- Equations Balance



Source: National Library of Virtual Manipulatives

- Hands-on Equations

Resources
Progression document for Grade 6- Grade 8 expressions and equations: https://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf Solving multi-step equations: http://illuminations.nctm.org/Activity.aspx?id=3482 : this website allows the students to solve multi step equations using algebra tiles. 7.EEI.2 - Balance Expressions - http://illuminations.nctm.org/Lesson.aspx?id=2747 7.EEI.2 - This website demonstrates how to derive the laws of exponents. Use this website as a guide for lesson planning. http://www.mathsisfun.com/algebra/exponent-laws.html 7.EEI.4 - This website provides a Jeopardy review game for students to practice the skills involved in solving multi-step equations and simplifying expressions. https://jeopardylabs.com/play/solving-multi-step-equations1
Sample Formative Assessment Tasks/Questions
7.EEI.1: The Martin family is remodeling their home. They are removing the wall between the kitchen and the dining room. Use the model below to determine the area of the new room.      Answer: 15x + 5 square units - Since the wall is being removed between the dining room and kitchen, students should add the lengths “x” and “2x+1” to determine the length of the newly formed room: 3x + 1 units. To determine the area, students must multiply the length by the width (or base by the height), so they should use the given width of 5 units, which is consistent for both the dining room and kitchen. By multiplying 5 by 3x + 1, it can be determined that the area of the newly formed room is 15x + 5 square units.

7.EEI.1: This task will assess students' knowledge of formulating an algebraic expression based on a real-life situation

The Quilt of a Math Teacher.pdf

Source: New York City Department of Education

7.EEI.2: This task has students become the business owner of a store and business is a little slow. Since a sale is the best way to attract customers, they have to determine which sale is best and justify their reasoning.

7.EE.A.2 Task Shop Smart.doc

7.EEI.3: In this task, students will use formulas to compute the number of points each player and team totals in an NFL Weekend for a fantasy football league.

Fantasy Football Task.pdf

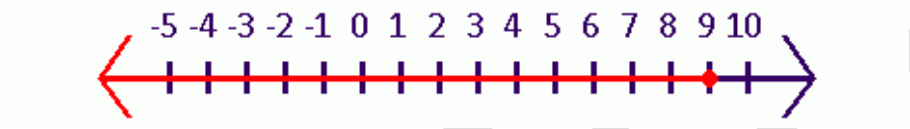
Source: Yummy Math

7.EEI.4: Thompsons Water Retreats rents boats for weekend trips. Each boat can carry 1742 pounds of people and luggage. The average weight of a person is 153 pounds, and each person brings a 40 pound suitcase.

Write an inequality to describe the restrictions on the number of people possible in a boat.

a. What is the maximum number of people that can rent a boat together?
b. Several families want to rent boats for July 4th. The first family has 4 people, the second has 11 people, and a third has 9. Which groups, if any, can rent a boat?
c. Construct a graph to represent the solution set.

Answers:

a. $x(153 + 40) \leq 1742$ or $193x \leq 1742$
b. 4 people - yes; 11 people - no; 9 people -yes - By solving for x, one gets the answer $x \leq 9.03$; this means total family members on the boat must be less than this number.
9 people - If $x \leq 9.03$, then a maximum of 9 people can be on the boat together.
c. 

7.EEI.4: This task challenges a student to use algebra to represent, analyze, and generalize a variety of functions including linear relationships.

Toy Trains.pdf

Source: Mathematics Assessment Resource Service

7.EEI.5: Solve the following expression: $\frac{3^0 \cdot (4^2)2}{2^2 \cdot 2^2}$

Answer: the solution to this question is 2. The first step would be to square 4 and get the value 16 and to rewrite the denominator as 2^4. Next would be to evaluate 3^0 as 1 and evaluate 2^4 as 16. Finally in the numerator multiply 16 and 2 to get 32 then divide 32 by 16 and get the final answer 2.

7.NS.4: Camille, the band director for the Middletown High School marching band, is buying instruments to expand the brass section. A tuba costs $960. The total expenditure must be below $2,700.

Write an inequality that describes this situation. Use the given numbers and the following variable.
x = the number of tubas purchased

Answer: $960x < 2700$

7.NS.4: Ning is printing out copies of a presentation. It takes 5 minutes to print a color copy. He wants to spend a maximum of 50 minutes on printing.

Write an inequality that describes this situation. Use the given numbers and the following variables.
x = the number of color copies

Answer: $5x \leq 50$

Unit Title
Ratios and Proportions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.RP.1 Compute unit rates, including those involving complex fractions, with like or different units. ○ Realize a complex fraction indicates a fraction within a fraction (e.g., $\frac{\frac{1}{2}}{\frac{1}{4}}$) **7.RP.2** Identify and model proportional relationships given multiple representations, including tables, graphs, equations, diagrams, verbal descriptions, and real-world situations. a. Determine when two quantities are in a proportional relationship. b. Recognize or compute the constant of proportionality. c. Understand that the constant of proportionality is the unit rate. d. Use equations to model proportional relationships. e. Investigate the graph of a proportional relationship and explain the meaning of specific points (e.g., origin, unit rate) in the context of the situation. ○ Distinguish between linear and proportional relationships. ○ Understand that proportional relationships are linear and include the origin in the solution set. **7.RP.3** Solve real-world and mathematical problems involving ratios and percentages using proportional reasoning (e.g., multi-step dimensional analysis, percent increase/decrease, tax). ○ Include sales tax, tip, markup/discount, simple interest, depreciation, commission, percent error ○ Solve for all values in a simple interest problem ○ Calculate the percent increase/decrease by $\frac{\lvert New-Original\rvert}{Original} = \frac{Part\ of\ Percent}{100}$ **7.NS.5** Extend prior knowledge to translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Exclude the conversion of repeating decimal numbers to fractions. ○ Recognize that $\frac{1}{3} = 0.\overline{3}\ and\ \frac{2}{3} = 0.\overline{66}$.

7.EEI.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

a. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where $a, b, and\ c$ are rational numbers.

b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides.

c. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.

d. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

- Include equations and inequalities with rational number coefficients, constants, and solutions.
- Justify steps and solutions using multiple means (e.g., substitution, properties, definitions)

7.GM.1 Determine the scale factor and translate between scale models and actual measurements (e.g., lengths, area) of real-world objects and geometric figures using proportional reasoning.

- Include examples for scaling up and scaling down (e.g., a scale model of an eye would be scaling up, a map would be an example of scaling down).

New Academic Vocabulary for This Unit

- Proportion
- Proportional relationships
- Complex fraction
- Constant of proportionality
- Like Terms
- Linear inequalities
- Scale factor
- Scale model

Prior Knowledge Required for this Unit
• Ratios, rates, and unit rates (6.RP.1, 6.RP.2, 6.RP.3, 6.EEI.9) • Expressions, equations, and inequalities (6.EEI.2b, 6.EEI.3, 6.EEI.4, 6.EEI.5, 6.EEI.7, 6.EEI.8, 6.EEI.9) • Translations among rational numbers (6.NS.9) • Scale factor of geometric shapes (6.GM.1)
Subsequent Knowledge Related to this Unit
In Grade 6, students determine the basic meaning ratios and unit rates, leading to complex ratios and rates in Grade 7. Subsequently, in Grade 8, students will apply unit rates to functions leading to an understanding of constant rate of change and slope. In high school mathematics courses, students will extend the knowledge of constant rate of change to include rate of change and average rate of change. In Grade 6, students solved problems involving one-step dimensional analysis. In Grade 7 and beyond, multi-step dimensional analysis is taught, used, and applied beyond mathematics courses (e.g., chemistry, physics). In Grade 6, students begin to translate among representations of rational numbers with limited denominators (2, 3, 4, 5, 8, 10, and 100), which is extended to all denominators in Grade 7. In Grade 8, students will extend their knowledge of translations among rational numbers to include the conversion of repeating decimal numbers to fractions. In Grade 6, students created and solved one-step equations and inequalities. In Grade 7, students create and solve multi-step linear equations and inequalities, utilizing the distributive property and combining like terms. Students extend knowledge of determining a solution set in Grade 6 to graph and interpret solutions to multi-step linear inequalities in Grade 7. In Grade 8, students solve linear equations and inequalities to include variables on both sides and systems of equations. In high school courses, students will create, solve, and graph linear and nonlinear equations and inequalities in two variables. Additionally, students will apply knowledge and understanding of solving equations to solve literal equations for an indicated value. In Grade 7, scale factor and scale model are introduced using proportional reasoning which leads to, in Grade 8, an understanding of the relationship between dilations and attributes of geometric figures. In high school, dilations will be extended to include negative scale factors, which represent reflections of enlargements and reductions.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop an understanding of proportional relationships and model those relationships using multiple representations.

Potential Instructional Strategies/Lessons

- **Real-world Problems**

Example 1:
If $\frac{1}{2}$gallon of paint covers $\frac{2}{5}$of a wall, then how much paint is needed for the entire wall?

Answer
½ gallon / ⅖ of a wall
1 ¼ gallons per wall.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 2:
The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship? Justify your reasoning.

Number of Books (x)	Price (y)
1	3
3	9
4	12
7	18

Answer
Students can examine the numbers to determine that the price is the number of books multiplied by 3,except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table therefore, the table does not represent a proportional relationship. Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the

table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books,0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when x= 1 will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 3:
Part A:
The price of bananas at Trader Joe's can be determined by the equation: P= $0.35n, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Part B:
Pete shopped for bananas at Publix on three separate occasions. The table below illustrates the prices Pete paid for the bananas. Determine the cost for 1 pound of bananas.

Number of Pounds of Bananas (x)	Total Price (y)
3 lb	$1.20
4 lb	$1.60
6 lb	$2.40

Answer
Part A: The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.
Part B: The price per pound of bananas is $0.40. The unit rate represents the constant of proportionality when the rate is represented by a denominator of 1.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 4:
Sally has a recipe that needs ¾ tablespoon of butter for every 2 cups of milk. If Sally decreases the amount of milk to 1 cup of milk, how many tablespoons of butter are needed?

Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

$$\frac{\frac{3}{4}}{2} = \frac{x}{1}$$

Answer
One possible solution is to recognize that $2 \div 2 = 1$ (or $2 \times \frac{1}{2}$) and that $\frac{3}{4} \div 2 = x$ (or $\frac{3}{4} \times \frac{1}{2}$). The amount of butter needed would be $\frac{3}{8}$ tablespoons.

A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \cdot 1 = 2x$. Solving for *x* would give $\frac{3}{8}$ tablespoons of butter.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 5:
Kohl's is having a 30% off sale on their sweaters. If the original price was $37.50, what is the sale price of the sweater before sales tax?

Answer
The discount is 30% of $37.50, or $11.25. The sale price of the sweater is the original price minus the discount, $37.50 - $11.25, or $26.25.
Alternately, complements can be used to calculate the sale price. If the discount is 30% off, the sale price is 70% of the original cost, 0.70 x $37.50.
Sale Price = 0.70 × Original Price.

Resources
7.GM.1 - This activity allows students to practice working with scale factors. http://illuminations.nctm.org/Activity.aspx?id=4207 7.EE1.4 - This game allows students an opportunity to graph simple linear inequalities. http://ltcconline.net/greenl/java/BasicAlgebra/inequalityGame/inequalities.html 7.EE1.4 - This game helps students understand the meaning of inequalities. http://www.xpmath.com/forums/arcade.php?do=play&gameid=87 7.RP.2 - This game allows an opportunity to solve proportions. http://www.xpmath.com/forums/arcade.php?do=play&gameid=97 7.EE1.4 - This game allows students to practice recognizing like terms. http://www.xpmath.com/forums/arcade.php?do=play&gameid=92 7.RP.2 - This activity checks for an understanding of constant of proportionality. http://www.mathopolis.com/questions/q.php?id=8943&site=1&ref=/algebra/directly-inversely-proportional.html&qs=8943_8945_8947_8948_8951_8952_7005_8064_8065_8067_8072 7.NS.5 - This game allows students to practice equivalences between decimals, fractions, and percents. http://www.mathplayground.com/Decention/Decention.html 7.RP.1 - This Jeopardy games gives students an opportunity to practice finding unit rates and make comparisons for the best price. http://www.quia.com/cb/195515.html

Sample Formative Assessment Tasks/Questions

7.RP.1: Jamal is painting his room. He determines that a $\frac{1}{2}$ gallon container of paint will cover $\frac{1}{6}$ of a wall. How much paint is needed for an entire wall (assuming there are no doors or windows)?

Answer
If a wall is divided into sixths, there will be six sections. Jamal will need 6 half-gallon containers; this is equivalent to 3 gallons of paint.

7.RP.2: The Smart Start Healthy Breakfast Company is studying the amount of sugar in common breakfast foods. Their findings are shown in the table below.

Breakfast Item	Serving Size (g)	Sugar (g)
Strawberry Pop-Tart	52g	16.5g
Cinnamon Toast Crunch Cereal Bar	45g	14g
KrispyKreme Glazed Doughnut	49g	10g

a. How many grams of Strawberry Pop-Tart will give you 22 grams of sugar? (Round your answer to the nearest whole number.)
b. Which breakfast food gives the highest ratio of sugar?

Answer
a. 66 grams
b. Strawberry Pop-Tarts have the highest ratio of sugar. (0.3173)

7.RP.2, 7.RP.3:

a. The total cost spent on baseball bats by the Columbia Fireflies when shopping at DICK's Sporting Goods can be determined by the equation: $C = \$196n$, where C is the price and n is the number of baseball bats purchased. What is the constant of proportionality (unit rate)?
b. The Fireflies also shopped for baseball bats at Academy Sports on four separate occasions. The table below illustrates the prices paid for the baseball bats. Determine the cost for 1 baseball bat.

Number of Baseball Bats Purchased (x)	Total Cost of Baseball Bats (y)
5	890
9	1602
11	1958
14	2492

c. Write an equation to represent the total cost (C) of buying n bats at Academy Sports.
d. Which store do you expect the Fireflies to use when purchasing baseball bats for the team? Justify your answer.
e. What is the markup (percent increase) used by the more expensive store? (Round to the nearest percent.)
f. Can you think of any reasons the Fireflies may choose to buy baseball bats from the more expensive store?

Answer

a. The constant of proportionality (unit rate) is \$196.
b. One bat will cost \$178 at Academy Sports.
c. $C = 178n$
d. Academy Sports; The Columbia Fireflies will spend less money per bat.
e. 10%
f. Student answers will vary but may include store location, bat quality, or contractual agreements.

7.RP.3: At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

Answer
The sales team members need to sell the original 48 plus an additional 30%. 100% of the TV sales is 48 while 30% is the number of TVs that need to be sold in addition to the 48. This leads to finding 130% of TVs sold to show an increase of 30% which is the same as multiplying 48 and 1.3 (130%) arriving at the answer of 63 TVs sold in May.

Alternate Solution: A second solution to this is to set up a proportion as a percent of increase question. $\frac{x}{48} = \frac{30}{100}$. After cross-multiplying to get the equation 100x=1440 then dividing by 100 to arrive at the answer 14.4. Since you cannot purchase a decimal portion of a TV you would need to round 14.4 to 15 TVs since we are looking for a full 30% increase in sales. Next would be to add the original 48 TVs and the 15 TVs increase in sales to a May.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

7.NS.5:
Complete the table with the appropriate fraction, decimal, or percent.

Fraction	Decimal	Percent
	0.83	
		32%
$\frac{22}{5}$		
		5.5%
	2.9	
		$\frac{3}{4}$%
$\frac{1}{6}$		

Answer

Fraction	Decimal	Percent
$\frac{83}{100}$	0.83	83%
$\frac{8}{25}$	0.32	32%
$\frac{22}{5}$	4.4	440%
$\frac{11}{200}$	0.055	5.5%
$2\frac{9}{10}$ or $\frac{29}{10}$	2.9	290%
$\frac{3}{400}$	0.0075	$\frac{3}{4}$%
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$%

7.EEI.4: Mr. Mann's class is going to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for rides and one for game booths. Each of the passes cost the same price.

a. Write an equation representing the cost of the trip.
b. Determine the price of one pass.

Answer

a. 52 = 2x + 11
b. Each pass costs $20.50.

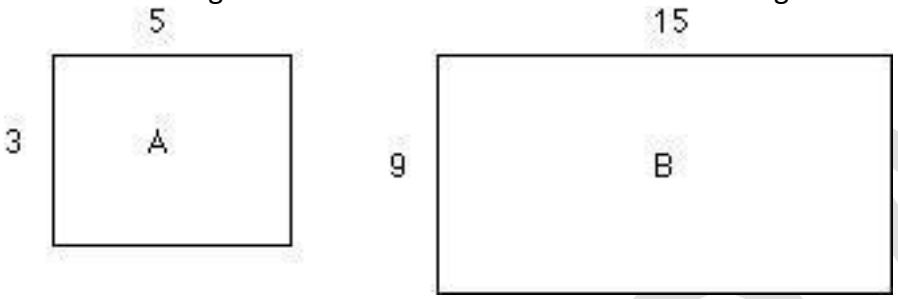
7.EEI.4: Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 and spend the rest on shirts. Each shirt costs $8.

a. Write an inequality for the number of t-shirts she can purchase.
b. Determine the number of shirts Florencia can purchase.

Answer

a. $60 > 22 + 8x$
b. $4.75 > x$, this means Florencia can buy 4 shirts.

7.GM.1: Rectangle B was reduced in size to become Rectangle A. What is the scale factor?

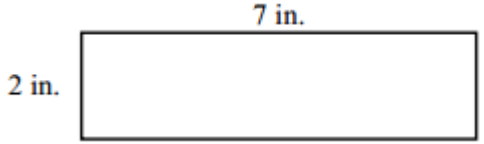


Answer

Set up a proportion $\frac{15}{9} = \frac{5}{3}$. Determine that you must divide 15 by 3 to get 5, therefore the scale factor is $\frac{1}{3}$.

Source: www.watertwon.k12.ma.us

7.GM.1: If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?



Perimeter _____

Area _____

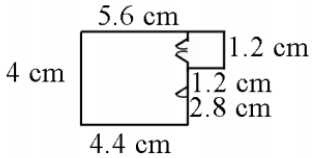
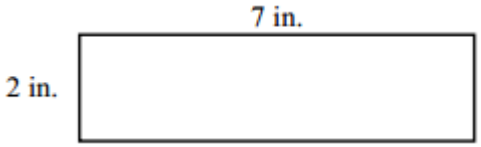
Answer
Perimeter = 27 *in.*
Area = 31.5 $in.^2$

Unit Title
Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.GM.1 Determine the scale factor and translate between scale models and actual measurements (e.g., lengths, area) of real-world objects and geometric figures using proportional reasoning. ○ Include examples for scaling up and scaling down (e.g., a scale model of an eye would be scaling up, a map would be an example of scaling down). ○ Make the distinction between a scale factor and scale. Scale factors are always expressed in the same units. (1cm to 50 cm) Scale is used when working with maps, drawings, or photographs and many times are expressed in two different units (1 inch = 10 miles). **7.GM.2** Construct triangles and special quadrilaterals using a variety of tools (e.g., freehand, ruler and protractor, technology). a. Construct triangles given all measurements of either angles or sides. b. Decide if the measurements determine a unique triangle, more than one triangle, or no triangle. c. Construct special quadrilaterals (i.e., kite, trapezoid, isosceles trapezoid, rhombus, parallelogram, rectangle) given specific parameters about angles or sides. ○ Two-dimensional figures that are constructed by freehand need not be drawn to scale. However, all measurements should be labeled. ○ Not all combinations of side measurements or angle measurements produce triangles. ○ The conditions for unique triangles are such that the dimensions will not satisfy another (unless they are congruent). ○ Some combinations of measurements will produce more than one triangle (For example, there are an infinite number of triangles that contain three 60°angles. Additionally, a triangle with side lengths 3 and 4 could have a third side that measures greater than 1 but less than 7.) ○ The lengths of any two sides of a triangle will have a sum greater than the third (Triangle Inequality Theorem). ○ Categorize triangles by the lengths of their sides (isosceles, equilateral, and scalene) as well as by the measure of their angles (right, obtuse, and acute). **7.GM.3** Describe two-dimensional cross-sections of three-dimensional figures, specifically right rectangular prisms and right rectangular pyramids. ○ Include horizontal, vertical, and diagonal cross-sections. ○ Predict the two-dimensional figures that result from slicing a 3-D figure such as a right-rectangular prism or pyramid. (perpendicular cuts and parallel cuts)

New Academic Vocabulary for This Unit
• Scale drawings • Scale factor • Cross-section • Freehand • Diagonal • Equiangular • Interior Angles

Prior Knowledge Required for this Unit
• Identify and describe two-dimensional shapes (K.G.2) • Identify triangles, quadrilaterals, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (2.G.1) • Understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories (3.G.1) • Identify a three-dimensional shape (i.e., right rectangular prism, right triangular prism, pyramid) based on a given two-dimensional net and explain the relationship between the shape and the net (3.G.4) • Recognize right triangles as a category, and identify right triangles. (4.G.3)
Subsequent Knowledge Related to this Unit
Constructions facilitate an understanding of geometry. Providing opportunities for students to construct angles, triangles, and quadrilaterals will allow students to discover the side and angle conditions that will form triangles. Students should be able to use this knowledge to determine if certain side lengths or angles can form a triangle and justify their conclusions with both sketches and reasoning. This leads directly into preparation for proofs in geometry. In high school mathematics courses, students will use their understanding of triangles' attributes to recognize similar and congruent triangles. Students will also draw conclusions about the perimeters, areas, and volumes of similar figures based on the scale factor that exists between them. Also in high school, students will extend their knowledge of cross sections to explain Cavalieri's Principle.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the attributes of triangles, quadrilaterals, and three-dimensional figures (e.g. prisms and pyramids). They also help develop 3-D visualization skills.

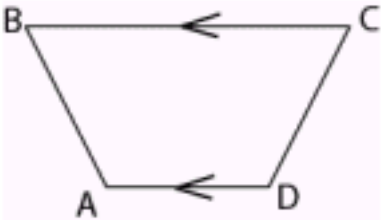
Potential Instructional Strategies/Lessons
• Dynamic Geometric Software (e.g. Geometer's Sketchpad or GeoGebra) • Compass and Straightedge construction • Play Doh and floss for cross sections - Have students form three-dimensional figures with the Play Doh and "slice" the figures using dental floss to determine horizontal, vertical, and diagonal cross sections. •  • Geometric Solids and Nets
Resources
7.GM.1 - This video discusses scale factors (including those greater than and less than one). https://learnzillion.com/lesson_plans/7167-generate-a-scale-drawing-using-scale-factors-greater-than-and-less-than-one 7.GM.1 - This YummyMath activity has students use proportional reasoning to discuss and theorize about the size of various television screens. http://www.yummymath.com/2015/glowing-rectangles-2/ 7.GM.2 - This video covers the construction of polygons based on given characteristics. https://www.youtube.com/watch?v=YB4FhIcCHro 7.GM.3 - This site defines a cross-section and also has an interactive activity which allows you to slice and see the cross-sections of 3D figures. http://www.learner.org/courses/learningmath/geometry/session9/part_c/

Sample Formative Assessment Tasks/Questions
7.GM.1: Julie shows the scale drawing of her room below. If 2 cm. on the scale drawing equals 5 ft., what are the actual dimensions of Julie's room?    *Answer* 5.6 cm. → 14 ft. 1.2 cm. → 3 ft. 2.8 cm. → 7 ft. 4.4 cm. →11 ft. 4 cm. → 10 ft. Source: NC DPI 7th Grade Mathematics Unpacked Contents
7.GM.1: If the rectangle below is enlarged using a scale factor of 1.5, determine the perimeter and area of the new rectangle.    Perimeter _____ Area _____ *Answer* The original rectangle measures 2 in. x 7 in. The scaled drawing would increase to the dimensions of 3 in. x 10.5 in. Thus, the new perimeter will be 27 inches and the area will be 31.5 in.^{2}.

7.GM.1: Maurice has a 60:1 scale-drawing of the floor plan for his new restaurant. His floor plan shows that the dimensions for his kitchen are $1\frac{2}{3}$ inches by $3\frac{1}{4}$ inches. What is the actual area of his kitchen in square inches? Is this the most reasonable unit of measure to use, if no, what would be better? *Answer* Using the ratio 60:1, we can determine that the actual kitchen is 100 inches by 195 inches. To determine the area in square inches we would multiply the two dimensions together and determine the area of the kitchen is 19,500 square inches. Square inches is not the best unit to use, square feet would have been a better option.
7.GM.2: Can a triangle have more than one obtuse angle? Explain your reasoning. *Answer* No, a triangle cannot have more than one obtuse angle because the interior angles of a triangle will have a sum of 180 degrees. If one angle is greater than 90 degrees, then the other two angles cannot have a sum to exceed 90 degrees. Therefore, there will only be one obtuse angle.
7.GM.2: Name the special quadrilateral given the parameters below: Base angles are 52∘ Non-parallel sides are 10 in and 15 in *Answer* The special quadrilateral that is formed is an isosceles trapezoid

7.GM.2: Draw a quadrilateral with one set of parallel sides and no right angles.

Answer
A trapezoid is a quadrilateral that has exactly one pair of parallel sides. This trapezoid does not have any right angles.



7.GM.2: Will three sides of any length create a triangle? Explain how you know which will work.
Possibilities to examine are:

a. 13 cm, 5 cm, and 6 cm
b. 3 cm, 3 cm, and 3 cm
c. 2 cm, 7 cm, 6 cm

Answer
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. If you take the three sides of a triangle and add them in pairs, the sum is greater than (not equal to) the third side. If this is not true, then it is not possible to construct a triangle with the given side lengths.

In the possibilities examined,

a. Based on the Triangle Inequality Theorem, the side lengths 13cm, 5cm, and 6cm, cannot create a triangle because the sum of 5cm and 6cm is 11cm which is not greater than the third side length of 13 cm.
b. Based on the Triangle Inequality Theorem, the side lengths of 3 cm, 3 cm, and 3 cm, can create a triangle. It would be considered an equilateral triangle due to all sides being equal.
c. Based on the Triangle Inequality Theorem, the side lengths of 2cm, 7cm, and 6 cm, can create a triangle. This triangle would be considered a scalene triangle, due to all unequal side lengths.

7.GM.3: Jordan is a master ninja and can slice straight through objects using nothing but his hands. Anxious to show off his skills, he approaches three boxes each in the shape of a cube. He slices through the first box horizontally, the second vertically, and the third diagonally. What two dimensional shape will each of the cross sections represent?

Answer
Horizontal: Square
Vertical: Square
Diagonal: Rectangle

Source: NC DPI 7th Grade Mathematics Unpacked Contents